Axial Dispersion of Spheres Fluidized with Liquids

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Solids dispersion due to the simultaneous diffusion (random particle motion) and classification (segregation) of fluidized spheres has been investigated. The model developed permits calculation of the mean concentration of spheres of both sizes in a mixture of two sizes of fluidized spheres as a function of bed length. Fick's law has been applied to the diffusional phenomenon. A hypothesis has been advanced and confirmed which permits calculation [Equation (5)] of the classification velocity for each size of sphere in a fluidized mixture. Calculation of the classification velocities in a mixture of spheres is based on the relation of void fraction to superficial velocity for the individual sizes of spheres. To facilitate carrying out these calculations, a generalized equation has been developed for the relation of void fraction to superficial velocity in terms of the Galileo number ($d^3 g \Delta\rho\rho_f/\mu^2$) and sphere-to-column diameter ratio. Measurement of the bead size gradients (change in bead size with bed length) at steady state fluidization was used to investigate dispersion. Closely sized glass beads of 0.1 and 0.2 cm. diameters were fluidized with liquids of 1 and 15 cps. at void fractions of 0.5 to 0.8 in 2.5 and 5.0 cm. columns. Lead beads of 0.12 cm. diameter were also used. Further, several experiments were conducted by a method which is analogous to that used in molecular diffusion cells.

The unique transport behavior which is observed in a fluidized bed is, of course, due to the mobility of the solid phase in addition to that of the fluid phase. The work reported here is an investigation of the diffusion and classification of spheres which have been fluidized with a liquid. *Diffusion* is used to describe the random mixing of the particles which is observed in fluidized beds. *Classification* refers to the tendency of a mixture of fluidized particles of different sizes or densities to separate into layers which are of the same size or density. These two mechanisms are the sources of dispersion which are considered here. As will be explained in more detail, since only the average concentration variations in the direction of fluid flow were measured, consideration is given only to dispersion in the axial direction.

PREVIOUS WORK

Richards (*1*) has shown in a remarkable photograph the importance of both classification and diffusion in the dispersion occurring in a fluidized bed of quartz and galena. Classification is clear in that the larger and denser particles tend to move to the bottom of the bed. Diffusion is observed since a wider range of particles is found in each section of the bed than would be possible if classification were not offset by mixing of the particles in the axial direction. In his work it is pointed out that the greater the terminal velocity of a single particle in an infinite media, the lower is the equilibrium level to which the particle tends to move, as a result of classification. This observation was observed regardless of the void fraction of the fluidized bed.

Brotz (2) used a method somewhat analogous to a molecular diffusion cell to measure the diffusion coefficient for the mixing of two colors of glass beads of the same diameter which were fluidized with water. For 0.2, 0.5, 0.8 and 1.0 cm. diameter beads at 1.3 to 2.3 times the superficial velocity of incipient fluidization, the diffusivity was found proportional to the bead diameter and the difference between the liquid superficial velocity at which the diffusivity was measured and the superficial velocity of incipient fluidization. A good deal of scatter was reported in the values of the diffusivities obtained. In essence, this work shows the applicability of Fick's law to the self-diffusion of fluidized spheres of a single size.

THEORY

A mathematical model is developed below which describes the axial dispersion of spheres which have been fluidized. The word *sphere* is used in its exact geometric sense. Consideration is given to a group of spheres (all of the same density) which is made up of one or more subgroups. All the spheres in any subgroup have the same diameter d_i. When there are two or more subgroups, classification tends to increase the concentration of the larger spheres in the bottom of the bed and to decrease the concentration of the same group at the top of the bed. The behavior of each of these groups is to be treated by the familiar continuum model of hydromechanics. Thus, the concentrations and velocities of each group are considered continuous functions in space and time. Since the axial dispersion of the spheres is being considered, the concentration is an integrated average over the cross-sectional area of the bed. It is to be expected that in the application of the continuum model to fluidized particles, more fluctuation in averaged quantities such as concentration and void fraction will be observed than with application to molecules, because of the much smaller number of the particles over which it is possible to average the measurements.

The concentration c_i of the spheres from the subgroup of size d_i or species i while fluidized is

$$c_i = w_i\,(1-\epsilon)\rho \qquad (1)$$

The continuum assumption means as applicable here that the concentrations, the weight fractions, and the void fraction are functions only of bed height x and time t. The total solids concentration c is

$$c = \sum_i c_i \doteq (1-\epsilon)\rho \qquad (2)$$

The group of spheres is a multicomponent mixture of i components. Each component has its own mass velocity

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V_i. It is advantageous to consider this velocity to be made up of three elements:

$$V_i = V + v_i + u_i \tag{3}$$

This divides the velocity of each subgroup into the average velocity V of the whole group of spheres and two velocities relative to this mean velocity: the classification velocity v_i and the diffusion velocity u_i. On the basis of Equation (3), the mass flux

$$j_i = c_i v_i + c_i u_i \tag{4}$$

of component d_i is defined relative to the mean mass flux of the spheres. The effects of the two mechanisms of dispersion which are being considered are each expressed in Equation (4) by a separate term.

The classification velocity v_i of species i in a mixture of spheres was introduced above; it is the mean rate at which particles tend to segregate based on differences in properties. The following relation for calculating its value when $V = 0$ (that is, batch operation) is proposed here

$$v_i = (U - U_i)/\epsilon \tag{5}$$

In Equation (5) the classification velocity for each species is so specified that the mean relative velocity of the spheres and the liquid is equal to its own fluidization velocity.

In Equation (5) U_i is the superficial velocity required to fluidize spheres of size d_i at the void fraction of the mixture in which the classification velocity is calculated, and U is the superficial velocity of the fluid. Since in general the void fraction varies up the fluidized bed, so will v_i. Spheres of size other than d_i influence the value of the classification velocity only as they contribute to the determination of the local void fraction as expressed by Equation (2).

A rationalization of the proposed method of evaluating the classification velocity follows. To maintain a group of spheres, made up solely of size d_i, at a specific void fraction ϵ_1 requires a superficial velocity of U_i. When the superficial velocity is only U spheres of size d_i fall at the rate indicated by Equation (5). For example, when hindered settling occurs, the superficial velocity is set by the displacement of the fluid by the solids and Equation (5) gives the settling velocity of the solids. The hypothesis advanced here is that Equation (5) applies to each group individually of a mixture of spheres of different sizes.

To evaluate the fluidizing velocity U_i, the following relation between void fraction and superficial velocity is used:

$$U_i = a_i \ \epsilon^{n_i} \tag{6}$$

As implied by this equation, the constants are different for each sphere size. To evaluate them the data of Richardson and Zaki (3) have been employed. To facilitate the evaluation of these constants a dimensionless group, referred to here as the Galileo number N_{Ga}, has been introduced.

$$N_{Ga_i} = \frac{d_i^3 \, g \, \Delta\rho\rho_f}{\mu^2} \tag{7}$$

The Galileo number contains all the pertinent physical properties of the solids and fluid and, together with the ratio of sphere to bed diameter, forms a basis for the correlation of the constants of Equation (6). Thus

$$a_i = \frac{\mu}{d_i \rho_f} \frac{\exp(-2.3\, d_i/D_b)}{\left(\dfrac{15.35}{N_{Ga_i}} + \dfrac{1.363}{{N_{Ga_i}}^{0.567}} \right)} \tag{8}$$

and

$$n_i = 5.57\, {N_{Ga_i}}^{-0.0779} + 28 \frac{d_i}{D_b} {N_{Ga_i}}^{-0.1186} \tag{9}$$

when

$$25 \leq N_{Ga} \leq 19000$$

and

$$0 \leq d/D_b \leq 0.04$$

Equations (8) and (9) permit evaluation of the fluidizing velocity by means of Equation (6) for a mixture whose void fraction is known. It should be noted that, when the void fraction is one and the bed diameter is infinite, Equations (6) and (8) give the fluidization velocity for a single sphere in an infinite medium. Furthermore, under these conditions, this is the relation between Reynolds number and Galileo number for a single sphere. While the Galileo number has appeared in the literature previously its importance has not been sufficiently emphasized.

Thus far, no restriction has been put on the number of subgroups considered. However, to simplify the development of the equations only two subgroups are considered below. That is, the group of fluidized spheres is considered a binary mixture made up of spheres of two sizes $\bar{d}_1$ and $\bar{d}_2$. As an extension of Brotz's (2) work, it is assumed that Fick's law applies to the diffusion of a binary mixture of spheres. Thus, the diffusion flux is determined by

$$c_i u_i = -D \frac{\partial c_i}{\partial x} \quad i = 1, 2 \tag{10}$$

It is assumed that the diffusivity D for the mixture of spheres is not a function of concentration

Consider a fluidized bed of fixed inventory (that is, $V = 0$) and at steady state (that is, $\partial c/\partial t = 0$); continuity of mass requires

$$j_i = 0 \tag{11}$$

The combination of Equations (2), (4), (5), (6), and (10) causes Equation (11) to be rewritten as

$$D \frac{dc_i}{dx} = c_i \left[\frac{U}{\epsilon} - a_i \left(1 - \frac{c_1 + c_2}{\rho} \right)^{n_i - 1} \right] \quad i = 1, 2 \tag{12}$$

For a known input of spheres of both sizes the average concentrations of each species is fixed. Thus

$$\bar{c}_i = \frac{1}{L} \int_0^L c_i \, dx \quad i = 1, 2 \tag{13}$$

are boundary conditions.

The four equations represented by (12) and (13) make up a simultaneous nonlinear first-order boundary value problem. The solutions to these equations required for comparison with experimental results were obtained with a digital computer with suitable numerical methods.

EXPERIMENTATION

Figure 1 is a schematic diagram of the apparatus used to investigate axial dispersion in fluidized beds. The heart of the apparatus was a precision glass column which was surrounded by a jacket assembled from sections of glass pipe. Columns of 1 and 2 in. diameter were used. Each column consisted of two halves each 23 in. long. The two halves could be aligned on their support so that no interruption of the flow occurred in passing from one half to the other. The joint between the two halves of the column was open about 0.013 in. A separator was strung with a layer of 0.008 in. nylon filaments which could pass into the column joint. The separator was moved into and out of the column joint by means of an air cylinder. The lower section of the column was equipped with a ⅛ in. thick sintered stainless steel grid which had a rated mean pore opening of 0.0165 cm. The grids were located about 5 in. below the column joint.

The liquid head in the column jacket served to prevent leakage of the liquid from the column joint under operating

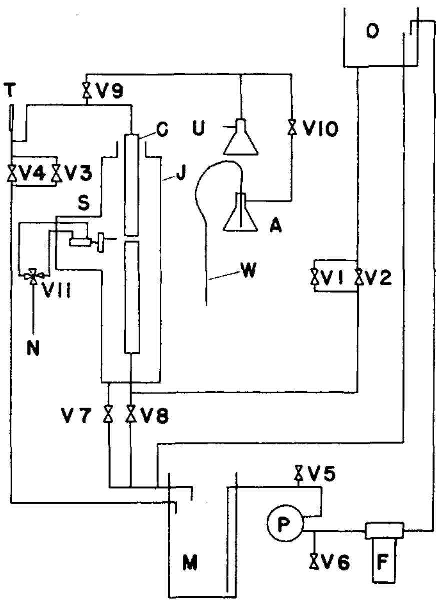

Fig. 1. Flow and control schematic diagram. A = vacuum cleaner system, C = column, F = filter, J = jacket, M = sump, N = nitrogen supply, O = constant level tank, P = pump, S = separator, T = thermometer, U = vacuum connection, W = wand.

conditions. By observing the changes in the liquid level in the jacket and adjusting the valves, it was possible to balance the flows in the two halves of the column.

This apparatus was operated to measure the axial solids concentration gradients in a fluidized bed. Two basic types of experiments referred to as transient and steady state experiments were made.

The transient experiments employed the same principle as that used in the diffusion cells for the measurement of the molecular diffusivity of a component of a fluid mixture. A vertical cylindrical bed was divided at its horizontal midplane by the separator. Glass beads, colored by gamma radiation, were placed above the separator and an equal weight of uncolored beads placed below it and both fractions of beads were expanded. The lower fraction of beads was expanded to the level of the bottom of the separator. When balanced flow rates were attained, the separator was moved laterally out of the column, and the two colors of beads diffused into one another. After a timed interval, the separator was returned to its original position in the column. The concentrations, bed section heights, and the time the separator was open were used to calculate the diffusion coefficient.

In the steady state experiments the decrease in the average bead size was determined for several bed sections up the column. This decrease in size up the bed, called a size gradient, is related to the classification and diffusion by the equations developed above. Since the variation in bead sizes was small for each lot of beads used, the effects of diffusion and classification were of comparable magnitudes.

In these latter experiments the beads were fluidized with the separator out of the column for 1 hr. at a constant void fraction. The equipment was operated so that the bed height was an integral multiple of the column length between the grid and the separator. After this period the bed was divided, usually in half, by means of the separator. The fractions into which the beads were divided were removed separately from the column. Two samples, each of five hundred-fifty beads, were taken from each fraction and the weight mean diameter of these samples were determined.

Several different lots of beads were used in the fluidization studies. The measured characteristics of these beads are shown in Table 1. The beads in lot 16 were lead; all the others were glass. These lots of beads were prepared for use by screening and, in some cases, fluidization to obtain a relatively small range of sizes in each lot. The beads were rolled down an inclined (about 0.5 in./ft.) plane of glass to remove grossly deformed and broken beads. The diameters of samples of fifty beads from lots 5 and 9 were determined with a microscope fitted with a ruled eyepiece. The diameter of samples of beads from the other lots were calculated from measurements of the individual beads and the lot density.[*] The sample size for these weighings was four hundred for lot 13 and two hundred for the remainder of the lots. The densities of the beads were determined by displacement of water.

TABLE 1. MEAN BEAD LOT MEASUREMENTS

Lot No.	Average diameter, cm.	Diameter standard deviation, cm.	Relative density, g./cc.	Two-group approximation*		
				$\bar{d}_1$, cm.	$\bar{d}_2$, cm.	f_1
5	0.0912	0.0059	2.862	—	—	—
9	0.0946	0.0053	2.862	—	—	—
13	0.0952	0.0032	2.862	0.09274	0.09789	0.5275
15	0.0973	0.0065	2.862	0.09167	0.1023	0.470
16	0.1222	0.0019	10.97	0.1205	0.1237	0.475
19	0.2053	0.0015	2.470	0.2040	0.2064	0.480

* Tabulated values are those actually used in calculations.

The model developed above is based upon a group of beads which are made up of two different sizes. The lots of beads which were used in experiments were made up of beads with a distribution of sizes and shapes. The distribution of sizes was approximated by two diameters for use in the calculations performed. This approximation is referred to as the "two-group approximation." The two-group approximation consists of three parameters ($\bar{d}_1$, $\bar{d}_2$, and f_1) which are based on the measured distribution of bead sizes used to determine the average diameter of the lot. The value of $\bar{d}_1$ is the average of the diameters of all the beads in the sample less than or equal to the average diameter for the whole lot. The value of f_1 is the ratio of the number of beads used to calculate $\bar{d}_1$ to the number of beads in the sample. The value of $\bar{d}_2$ is the average of the diameters of the beads in the sample larger than the average diameter for the whole lot. The two-group approximation values are included in Table 1 for those lots which were used for steady state runs.

Distilled water and a distilled water solution of propylene glycol were used to fluidize the beads. The viscosity of the glycol solution ranged from 14 to 16 centipoise for the reported runs.

RESULTS AND DISCUSSION

Transient Experiments

The results of the transient runs are presented in Table 2. It was observed during runs 7 and 12 that classi-

TABLE 2. SUMMARY OF RESULTS FOR TRANSIENT RUNS

Run	7	12	44
Bead lot	5	9	19
Liquid	water	water	glycol
Column diameter, cm.	2.537	2.537	5.076
Mean void fraction	0.511	0.513	0.497
Superficial velocity, cm./sec.	2.09	2.06	0.543
Diffusion time, sec.	51.8	49.2	285.3
Ave. rel. conc., sect. 1*	0.69	0.76	0.75
Diffusivity, sq. cm./sec.	0.96	0.60	0.112
Galileo number	18,700	17,900	460

* Section 1 is bottom section.

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[*] A table showing the distribution of bead sizes has been deposited as document 8565 with the American Documentation Institute, Photoduplication Service, Library of Congress, Washington 25, D. C., and may be obtained for $1.25 for photoprints or 35-mm. microfilm.

fication of the beads occurred while the flows were being adjusted. Thus, when the separator was opened, there were smaller beads at the top of section one which diffused into larger beads at the bottom of section two. To some degree, dispersion of beads between the two sections of the column by diffusion was increased by classification in all the transient runs. This effect appears to have been small.

Steady State Experiments

Most of the experimental results are from steady state runs. Experiments and computations indicate that sufficient time was allowed in all of these runs to obtain at least 99% of the change in bead concentration which could occur between the initial condition and steady state. Table 3 gives the measured bead sizes for the steady state runs. It is apparent that the beads from section one have a greater diameter than those from section two. This size gradient, due to classification, was observed to exist and to be significant in all the steady state runs. The size gradient was found by statistical test to be independent of void fraction for a given bed length and bead lot.

For bead lot 13 there is a statistically significant difference between replications made with the same fluid and column diameter at the 95% level. This is the type of variation which would result from "statistical" fluctuations about the mean size gradient. The variation in the size gradient was not significant for other bead lots, different fluids, or different column diameters.

A comparison of the results obtained with different bead lots is shown in Figure 2. All the two section runs for each bead lot have been averaged to obtain the results presented in this figure. To allow for differences between the average diameters of the lots, a dimensionless basis has been used. The ratio of the difference between the weight mean diameters of the two bed sections and the difference in the two-group approximation diameters is called the separation factor. The separation factor is approximately a normalized measure of the amount of separation by classification which has occurred during fluidization. Consideration of the definition of the two group approximation shows that if complete classification occurred, the separation factor is approximately one. If no classification occurs, the diameters in both sections are equal and the separation factor is zero. The standard deviation of the distribution of beads in the lot divided by the average diameter or the reduced standard deviation is the quantity used to compare the diameter variation observed in each bead lot. The separation factor is seen to decrease linearly with the reduced standard deviation. The difference in the two group approximation diameters is roughly proportional to the standard deviation. Therefore, the gradient is proportional to the square of the standard deviation for the lot of beads.

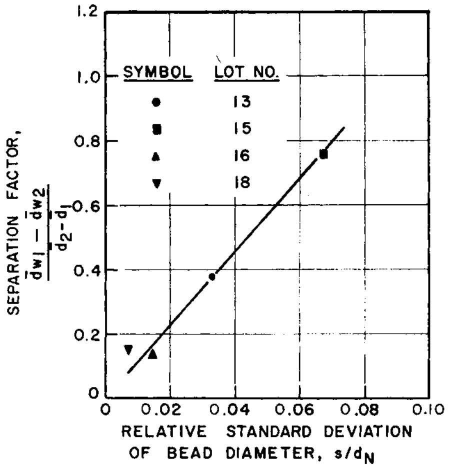

Fig. 2. Comparison of average results for steady state runs with different bead lots (L = 25.5 cm.).

TABLE 3.* RESULTS OF CALCULATIONS FOR STEADY STATE RUNS

Run	F-Lt-C	Void fraction	N_{Ga}	$\bar{d}_w$, cm.	D, sq. cm./sec. Approx.	Computer
19	$W13s$	0.798	21,200	0.0960 0.0944	4.38	4.46 4.34
20	$W13s$	0.509	20,700	0.0964 0.0941	1.56	1.46 1.42
22	$W13s$	0.699	20,700	0.0963 0.0941	2.58	2.58 2.25
24	$W15s$	0.799	25,000	0.1009 0.0936	3.80	3.46 1.72
27	$W13s$	0.799	21,200	0.0967 0.0954 0.0940	5.55	4.68 — 5.75
29	$G13s$	0.510	80	0.0960 0.0946	0.403	0.378 0.475
30	$G13s$	0.800	75	0.0970 0.0937	0.730	0.480 0.615
32	$G16s$	0.511	970	0.1223 0.1218	2.23	
33	$G16s$	0.588	830	0.1222 0.1218	3.80	
35	$G15s$	0.510	78	0.1018 0.0939	0.268	
36	$G15L$	0.510	65	0.1016 0.0931	0.316	
37	$G15L$	0.799	90	0.1014 0.0930	1.35	
41	$G19L$	0.498	510	0.2056 0.2052	0.399	0.220 1.51
43	$G19L$	0.750	530	0.2057 0.2052	0.941	

* A more complete table has been deposited with the American Documentation Institute. See footnote on page 26.

Note: d and D are tabulated in order of increasing bed section number down the table.

Lt = bead lot No.; F = fluid used; W = water; G = glycol; C = column diameter; s = 2.537 cm.; L = 5.076 cm.; $\bar{d}_w$ = weight mean diameter; D = diffusivity, sq. cm./sec.; N_{Ga} = Galileo number.

For the two runs made with three bed sections with the use of bead lot 13, the mean bead diameter decreased linearly with bed height. For the two and three bed section runs the size gradient was inversely proportional to the total bed height.

Calculations

The solution to Equations (12) and (13) is the concentration of each of the two groups of beads determined as a function of bed height. A typical solution is plotted in Figure 3. This solution was obtained by using the Runge-Kutta method on a digital computer.

Calculations were made which were based upon the steady state runs in order to determine the effectiveness of the model developed above. As will be seen in the following summary of the calculations, several additional assumptions were required to carry out the calculations. The length and diameter of the bed as well as the liquid density and viscosity were determined for each run. The bead lot employed determined the bead density and the three parameters of the two-group approximation ($\bar{d}_1$, $\bar{d}_2$, f_1) which were discussed above. These data permit calculation of the constants (a_i, n_i) of the relation of void fraction to superficial velocity with Equations (8) and (9) for both the bead sizes of the two-group approximation. Now the classification velocity of Equation (5)

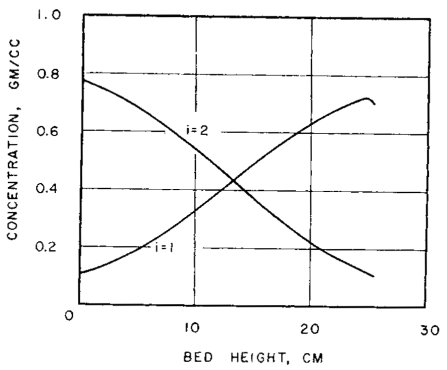

Fig. 3. Calculated sphere concentrations vs. bed height. (Based on run 18 made with bead lot 13. Calculations made with D = 2.49 sq.cm./sec., Δx = 0.27 cm., L = 25.54 cm. Mean void fraction = 0.699, water temperature = 80°F.).

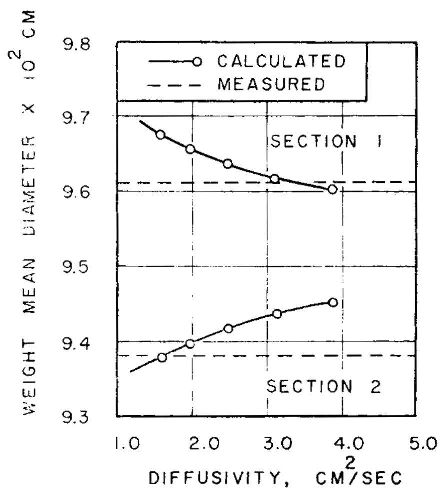

Fig. 4. Calculated weight mean diameter vs. diffusivity based on run 18.

was found to be the difference between two numbers of similar magnitude. Therefore, the moderate deviation which existed between the observed and the calculated superficial velocity caused large errors in the classification velocity when the experimentally measured superficial velocity was used. Therefore, a calculated value of superficial velocity was used for evaluating the classification velocity. This value was determined with Equation (6) by use of the mean void fraction for the whole bed which was measured during the run and constants determined for the average bead diameter of the lot. The value of the bead diffusivity D was established before the calculations were started. The mean concentration of each species in the fluidized bed was calculated from the mean void fraction and the bead properties. In the manner just described, all the constants required to specify the solution to Equations (12) and (13) were determined.

As mentioned above, numerical methods were employed to obtain actual solutions. This method necessitated us to assume a bed height increment and the concentration of each species at the top of the bed and then to integrate over the length of the bed. If the boundary conditions specified by Equation (13) were not satisfied to within 0.0003 g./cc. new estimates were made of the concentrations at the top of the bed and the procedure was repeated. A modification of the Newton-Raphson method (*4*) was used to make these estimates.

Calculations were made for different diffusivities but otherwise identical conditions. From the values of sphere concentrations which were calculated as a function of bed height, values of the mean sphere diameters were calculated. These diameters were calculated for the sections of the bed corresponding to the sections for which mean diameters were measured in the experimental runs. Figure 4 shows typical calculated mean diameters as a function of the corresponding assumed diffusivity for otherwise identical conditions. The weight mean bead diameters for the corresponding experimental run are also indicated in this figure. The diffusivity which corresponds to the experimentally measured bead diameter on such a graph is referred to as the computer value. Thus, for run 18 the computer diffusivities determined for bed sections one and two are 3.4 and 1.6 sq. cm./sec., respectively. The precision of the values of the diffusivity determined in this way depends primarily on the diameter measured for the bed section and the two-group approximation diameters measured for the bead lot. Based on these two sources of variation, it has been calculated that the estimated error is such that the true value of the diffusivity for run 18, section 1, is between 2.1 and 5.0 sq. cm./sec. Similar precision is associated with all measurements made with bead lot 13. Thus, while the precision of the diameter measurements was about ±0.4%, the precision of the diffusivity measurements was about ±40% for bead lot 13. The calculated values of diffusivity are presented in Table 3. What has been termed the statistical fluctuation in the bead size gradient also causes a fluctuation in the calculated diffusivities. This fluctuation prevents a definite correlation of the diffusivities. However, it was observed that the bead diffusivity was nearly proportional to the liquid superficial velocity.

Computer diffusivities were obtained for both the top and bottom sections of the bed for runs (including three section runs) made with bead lot 13. Further, computer diffusivities were obtained for one of the runs made with each of the lots 15, 16, and 19. Also calculated for each steady state run was a value of diffusivity, the approximate diffusivity, from an approximate equation presented in the Appendix. The average of the two computer diffusivities for a run have been plotted vs. the corresponding approximate diffusivity in Figure 5 when possible. The average deviation between diffusivity values calculated by the two methods for lot 13 is about 10%. The agreement observed between the two methods and the higher variation in diffusivity obtained for runs with bead lots 15, 16, and 19 led to the decision not to perform further calculations to obtain computer diffusivities for the other runs.

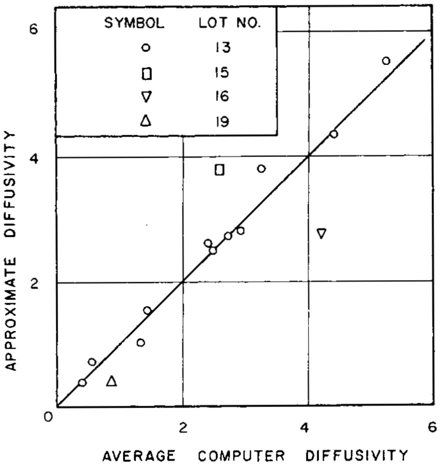

Fig. 5. Comparison of approximate and computer diffusivities.

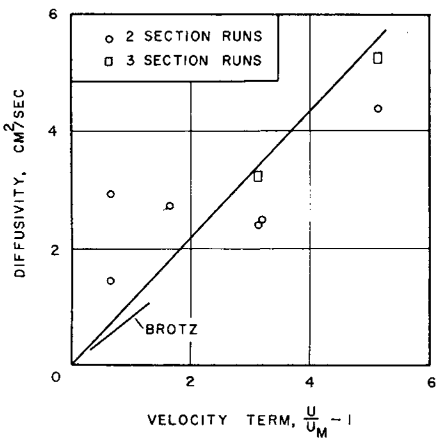

Fig. 6. Comparison of steady state run results of this work and those of reference 2.

It will be observed in Figure 3 that the concentration of the smaller spheres ($i = 1$) reaches a maximum near the top of the bed. This maximum, called a hook, was observed in practically all cases. Consideration of Equations (6) and (12) shows that the total sphere concentration above the maximum in the hook is less than it would be for either size of bead fluidized alone at the same superficial liquid velocity. It is believed that this does not occur in the fluidized beds, although this point has not been specifically investigated. The amplitude of the hook (maximum concentration minus the concentration at the top of the bed) decreased as the range of the bead lot sizes decreased.

The bed height increments used in the calculations were from 0.03 to 0.5 cm. depending on the run. Calculations were made for fixed conditions with different bed height increments. These calculations showed that the solutions have converged for the results reported here.

Experiments and Calculations Compared

The calculated results are in general agreement with the experimentally observed facts. Figure 3 shows that the large spheres concentrate at the bottom of the bed, while the smaller spheres concentrate at the top of the bed as a result of classification. Both experiments and calculations show that when the beads used have a wider size range (but are otherwise similar) there is a greater size separation due to classification.

Values of diffusivity measured in the *transient* runs (Table 2) were used together with the model developed above to calculate the mean bead diameter in each of the bed sections for three similar *steady state* runs. These calculated diameters are compared to the corresponding measured values in Table 4. The factors which are important in judging the significance of the differences observed in Table 4 are: the use of the two-group approximation, the difference between bead lot 13 and lots 5 and 9, the precision of the measurements of the diameters and diffusivity, and the statistical fluctuation. In view of these factors, it is believed unlikely that the differences observed between the calculated and measured diameters in Table 4 are significant.

TABLE 4. COMPARISON OF EXPERIMENTAL AND CALCULATED BEAD DIAMETERS
(Calculations are based on proposed model with the use of independently determined diffusivities)
Void fraction 0.50 to 0.51

Steady state run No.	20	28	41
Bead diameter, cm.			
Bed section 1			
Experimental	0.0964	0.0960	0.2056
Calculated with			
$D = 0.112$			0.2058
$D = 0.60$	0.0972	(0.0972)	
$D = 0.96$	0.0968	(0.0968)	
Bed section 2			
Experimental	0.0941	0.0946	0.2052
Calculated with			
$D = 0.112$			0.2047
$D = 0.600$	0.0934	(0.0934)	
$D = 0.960$	0.0938	(0.0938)	

Note: Values in brackets are estimated from calculations for run 20.

TABLE 5. COMPARISON OF EXPERIMENTAL AND CALCULATED VOID FRACTIONS FOR RUN 18
($D = 2.5 \pm 1$)

Source	Void fraction in bed Section 1	Section 2
Experimental	0.698 ± 0.001	0.700 ± 0.002
Calculated with $D = 3.88$	0.697	0.701
Calculated with $D = 1.59$	0.695	0.703

TABLE 6. A COMPARISON OF TRANSIENT AND BROTZ'S RESULTS

Run No.	Diffusivity, sq. cm./sec. Experimental	Brotz calculation
7	0.96	0.55
12	0.60	0.57

The void fraction in each section of the bed for each run was calculated and was measured in the course of the experimental investigation. The values for a typical run are compared in Table 5. The experimental and calculated values agree within the limits of the precision of the measurements.

It was observed that some calculations have been made beyond the limits of Equations (8) and (9). This extrapolation has been checked as carefully as the meager data permits. The additional error due to this extrapolation appears to be negligible.

Comparison with Previous Work

The results of Brotz (*2*) were obtained by a method which is most comparable to the transient experiments of this investigation. The two transient runs made with water are compared in Table 6 to values obtained by extrapolation from Brotz's results. The differences do not appear to be significant. Extrapolated results of Brotz are also compared to the diffusivities from relevant steady state experiments in Figure 6. A line representing the data has been drawn through the origin in recognition of the expectation that the diffusivity is zero for that condition. The values indicated for Brotz's work represent a linear extrapolation of his values from beads of 0.2 cm. and larger in diameter to about 0.1 cm. The agreement is good.

The experimental results are qualitatively in agreement with the work of Richards (*1*).

CONCLUSIONS

The axial dispersion occurring in a batch of beads fluidized with a liquid results in the characteristics enumerated below. The effect of each parameter on the bead size gradient is considered with the others constant. Of course, these conclusions are limited to the range of this investigation.

1. The mean bead size gradient is inversely proportional to the bed length.

2. For closely sized beads, the mean bead size gradient is proportional to the square of the standard deviation of the bead size distribution.

3. At steady state fluidization the bead size gradient fluctuates randomly with time about the mean bead size gradient.

4. The mean bead size gradient is independent of the bed void fraction, the liquid viscosity, the bead density and the bed diameter.

It is clear the fluctuation in the mean size gradient may obscure the influence of parameters which have a small effect on the mean size gradient.

The model developed above for the dispersion of spheres and the approximations used for a distribution of bead sizes are effective in predicting the mean size gradient from fluidized beads. The hook, discussed above, is due to the assumed relation between the two-group approximation and the calculated superficial liquid velocity. Equation (5) gives the classification velocities in mixtures of fluidized spheres.

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NOTATION

a_i = constant in void velocity relation, L/t
c = concentration of fluidized spheres, M/L^3
c_i = concentration of spheres of size d_i, M/L^3
$\bar{c}_i$ = mean concentration of c_i in bed length of L, M/L^3
d_i = diameter of spheres of species, i, L
$\bar{d}_w$ = weight mean diameter, L
$\bar{d}_1, \bar{d}_2$ = diameters of the two-group approximation, L
D = diffusion coefficient for fluidized spheres, L^2/t
D_b = diameter of fluidized bed, L
f_1 = fraction of beads in the two-group approximation of size $\bar{d}_1$, dimensionless
F_i = fraction of beads in a group of size d_i, dimensionless
g = acceleration of gravity, L/t^2
j_i = mass flux of beads of size d_i, M/L^2t
L = height of the fluidized bed, L
n_i = exponent in void velocity relation, dimensionless
N_{Ga} = Galileo number, Equation (7), dimensionless
s = standard deviation of bead diameter, L
u_i = diffusion velocity of spheres of size d_i (center of mass), L/t
U_i = superficial velocity required to maintain spheres of size d_i at a void fraction ϵ, L/t
U = superficial velocity of liquid through bed, L/t
U_m = superficial velocity at incipient fluidization, L/t
v_i = classification velocity (center of mass) for spheres of size d_i, L/t
$\bar{v}_i$ = classification velocity for spheres of size d_i at the mean void fraction of bed, L/t
V = mean velocity of spheres with respect to the tube, L/t
V_i = mean velocity of spheres of size d_i, L/t
w_i = weight fraction of spheres of size d_i, dimensionless
$\bar{w}_i$ = weight fraction of beads of size d_i in the bed, dimensionless
Δw_i = difference between mean weight fractions of spheres of size d_i in bed sections 1 and 2, dimensionless
x = bed height coordinate, L

Greek Letters

ϵ = void fraction at a position in the bed, dimensionless
μ = liquid viscosity, M/Lt
ρ = bead density, M/L^3
ρ_f = fluid density, M/L^3
$\Delta\rho$ = $\rho - \rho_f$, M/L^3
Σ_i = summation over all i

Subscript

i = variables dependent on sphere diameter d_i

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APPENDIX

The basis for the calculation of the approximate value of diffusivity is developed here. Consider a group of spheres of uniform density made up of two subgroups, each of uniform diameter, $\bar{d}_1$ and $\bar{d}_2$ respectively. The number fraction of spheres of size $\bar{d}_1$ is F_1 and of $\bar{d}_2$ is F_2. The weight mean diameter of the group is designated $\bar{d}_w$ and defined by

$$\bar{d}_w^3 = F_1 \bar{d}_1^3 + F_2 \bar{d}_2^3 \tag{A1}$$

The weight fraction of spheres in the group of size d_1 is designated w_1 and given by

$$w_1 = F_1 \bar{d}_1^3/(F_1 \bar{d}_1^3 + F_2 \bar{d}_2^3) \tag{A2}$$

There is a corresponding equation for the weight fraction of spheres of the other size. Now, if the number fractions are eliminated from equations (A1) and (A2), the weight fraction is

$$w_1 = \frac{(\bar{d}_2/\bar{d}_w)^3 - 1}{(\bar{d}_2/\bar{d}_1)^3 - 1} \tag{A3}$$

Now, if the values of $\bar{d}_1$ and $\bar{d}_2$ are known, it is possible to calculate the weight fraction of beads of size $\bar{d}_1$ from a measured value of $\bar{d}_w$ for a sample of beads which has been classified.

An approximate form of Equation (12) will be derived. The derivative term can be expressed in terms of the difference between the average concentrations of the two bed sections and the bed length as

$$\frac{dc_i}{dx} \approx \frac{\Delta c_i}{L/2} \tag{A4}$$

By the use of the mean concentration of spheres for the whole bed and the classification velocity at this concentration in Equation (12) together with Equation (A4), the diffusivity is approximately

$$D \approx \frac{L\,\bar{c}_i\,\bar{v}_i}{2\Delta c_i} \tag{A5}$$

By assuming the difference in void fractions between bed sections is negligible one can apply Equation (1) to each concentration in Equation (A5) to obtain

$$D = \frac{L\,\bar{w}_i\,\bar{v}_i}{2\Delta w_i} \tag{A6}$$

Equation (A6) was used with Equation (A3) and the two-group approximation values to calculate approximate values of diffusivity for the steady state runs.

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